A model realisation of the Jaffe-Wilczek correlation for pentaquarks

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Abstract

We discuss a realisation of the pentaquark structure proposed by Jaffe and Wilczek within a simple quark model with colour-spin contact interactions and coloured harmonic confinement, which accurately describes the $\Delta - N$ splitting. In this model spatially compact diquarks are formed in the pentaquark but no such compact object exists in the nucleon. The colour-spin attraction brings the Jaffe-Wilczek-like state down to a low mass, compatible with the experimental observation and below that of the naive ground state with all S-waves. We find, however, that although these trends are maintained, the extreme effects observed do not survive the required "smearing" of the delta function contact interaction. We also demonstrate the weakness of the "schematic" approximation when applied to a system containing a P-wave. An estimate of the anti-charmed pentaquark mass is made which is in line with the Jaffe-Wilczek prediction and significantly less than the value reported by the H1 collaboration.

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1 Introduction

In the wake of the possible discovery of baryon with positive strangeness (the Θ) [1], phenomenological models have arisen designed to explain its low mass ($\sim$ 1540MeV) and narrow width ($<$ 10MeV). For an early review see [2].

One such model is due to Jaffe and Wilczek [3]. They propose diquark correlations of low mass in a relative P-wave, giving the state overall positive parity in line with the Chiral Soliton Model prediction [4].

The Jaffe-Wilczek model has two diquarks each of spin-0, flavour $\bar{\mathbf{3}}_F$ and colour $\bar{\mathbf{3}}_c$, which by Bose statistics must couple symmetrically. To give an overall $\overline{\mathbf{10}}_F$ the diquarks must couple to a $\bar{\mathbf{6}}_F$, i.e. symmetrically. An overall colour singlet requires antisymmetric diquark coupling to a $\mathbf{3}_c$ and hence to be overall symmetric we are forced to introduce a P-wave between the diquarks. A P-wave appears in many quark model treatments of pentaquark structure, but, unlike in the Jaffe-Wilczek model, it is usually rather *ad hoc*.

This model has attracted much attention, and is particularly appealing as the same diquark correlations can be used to explain the enigmatic scalar mesons below 1GeV as being diquark-antidiquark states [5, 6].

Our point of departure is to discuss the dynamics which gives rise to diquarks. This an open question which must eventually be answered directly from QCD, but we will show that such correlations can appear even within simple quark potential models of the type often used to describe the conventional hadron spectrum.

2 Schematic Approximation, Quark Potential Model and a Jaffe-Wilczek-like state

Previous investigations into pentaquark structure have often made use of the "Schematic" approximation to colour-spin forces [7]. This approximation discards spatial dependence, having an interaction potential $V^{\sigma}_{\rm SCH} = -C\sum_{i,j}\frac{\vec{\lambda}_i}{2}\cdot\frac{\vec{\lambda}_j}{2}\vec{S}_i.\vec{S}_j$, where C is a constant. For example in the total spin-0 channel this would give,

$$\langle\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_c|V^{\sigma}_{\rm SCH}|\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_c\rangle = \begin{array}{c} -C \\ 0 \end{array} \left\{ \begin{array}{l} 0\otimes 0\to 0 \\ 1\otimes 1\to 0 \end{array} \right. , \tag{1}$$

where e.g. $1\otimes 1\to 0$ indicates that 1 & 2 and 3 & 4 are each coupled to spin-1 and then the two pairs coupled to total spin-0.

We will go beyond this approximation by allowing there to be a non-trivial spatial dependence. In particular we shall see that the schematic approximation, while capturing all the physics for spatially symmetric states, is not sufficiently versatile to accurately describe states containing a P-wave. This will be demonstrated by computing the equivalent to (1) in a model with non-trivial spatial dependence.

We now introduce the model which, in essence, is a standard quark potential model suitable for describing the light baryon and meson spectrum.

As a binding potential between quarks we take the coloured harmonic oscillator,

$$V(q_iq_j) = -a\frac{\vec{\lambda}_i}{2}\cdot\frac{\vec{\lambda}_j}{2}\,(\vec{r}_i-\vec{r}_j)^2. \tag{2}$$

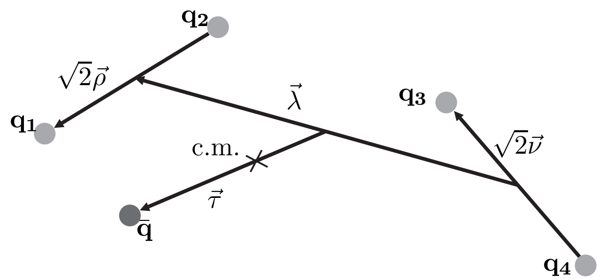

Figure 1: Five-body co-ordinate set in the $q^4\bar{q}$ rest frame.

This is chosen mainly for ease of calculation but has been used in the past as an approximation to the more phenomenologically justified coloured Coulomb + linear potential.

In addition to this we introduce a colour-spin contact interaction,

$$V^{\sigma}(q_i q_j) = -h\frac{\vec{\lambda}_i}{2}\cdot\frac{\vec{\lambda}_j}{2}\,\vec{S}_i.\vec{S}_j\,\delta(\vec{r}_{ij}). \tag{3}$$

It is this potential that will act to bind two quarks tightly into an effective light diquark.

The fundamental mechanism binding the diquarks in the Jaffe-Wilczek picture is unlikely to be anything so simple as a colour-spin contact interaction, and indeed it may not be possible to consider diquarks in terms of constituent quarks with non-relativistic interactions at all. Diquarks may appear as degrees-of-freedom in the same way as constituent quarks do during chiral symmetry breaking. Despite this, colour-spin contact interactions have a considerable phenomenological pedigree and may well be able to simulate quite effectively the true QCD interaction responsible for the “fine” structure in hadrons and as such we will investigate their effect on the pentaquark system. The motivation we have for a delta-function interaction is that of [8], the Breit-Fermi reduction of one-gluon exchange between quarks, but delta-function interactions have also been used to model instanton effects in light hadrons (see for example [9] and references therein).

Working in the $q^4\bar{q}$ rest frame, we define the internal variables, $\vec{\tau},\vec{\lambda},\vec{\rho},\vec{\nu}$, by

$$\begin{aligned}
\vec{r}_1 &= -\frac{\mu}{4m}\vec{\tau}+\frac{1}{2}\vec{\lambda}+\frac{1}{\sqrt{2}}\vec{\rho}\\
\vec{r}_2 &= -\frac{\mu}{4m}\vec{\tau}+\frac{1}{2}\vec{\lambda}-\frac{1}{\sqrt{2}}\vec{\rho}\\
\vec{r}_3 &= -\frac{\mu}{4m}\vec{\tau}-\frac{1}{2}\vec{\lambda}+\frac{1}{\sqrt{2}}\vec{\nu}\\
\vec{r}_4 &= -\frac{\mu}{4m}\vec{\tau}-\frac{1}{2}\vec{\lambda}-\frac{1}{\sqrt{2}}\vec{\nu}\\
\vec{r}_{\bar{q}} &= \frac{\mu}{m_{\bar{q}}}\vec{\tau},
\end{aligned} \tag{4}$$

which are clearly well suited to describing a Jaffe-Wilczek-like configuration if the diquarks are

spatially correlated (and see figure 1). With these variables the kinetic energy is[2]

$$T(q^4\bar{q}) = \frac{\vec{\nabla}_\tau^2}{2\mu} + \frac{\vec{\nabla}_\lambda^2}{2m} + \frac{\vec{\nabla}_\rho^2}{2m} + \frac{\vec{\nabla}_\nu^2}{2m}, \tag{5}$$

and we can write the Hamiltonian $H(q^4\bar{q}) = T(q^4\bar{q}) + V(q^4) + V(\bar{q}) + V^\sigma(q^4) + V^\sigma(\bar{q})$.

We can approximately solve for an eigenstate of this Hamiltonian using the variational method. First introduce a trial wavefunction containing a number of variational parameters and minimize the expectation value of the Hamiltonian with respect to them. The trial wavefunction has spatial form

$$\psi_m = N[\beta\lambda Y_{1m}(\hat{\lambda})e^{-\beta^2\lambda^2/2}][Y_{00}(\hat{\rho})e^{-\gamma^2\rho^2/2}][Y_{00}(\hat{\nu})e^{-\gamma^2\nu^2/2}][Y_{00}(\hat{\tau})e^{-\alpha^2\tau^2/2}], \tag{6}$$

which is a P-wave in the $\vec{\lambda}$ variable and an S-wave in the others in the harmonic oscillator approximation. The flavour-spin-colour structure is chosen to be $|\bar{\mathbf{3}}_0, \bar{\mathbf{3}}_0\rangle_{\bar{\mathbf{6}},0} \otimes |\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c\rangle_{\mathbf{3}_c}$, where this is a shorthand for quarks 1 & 2 coupled to a $\bar{\mathbf{3}}$ of flavour, spin 0 and colour $\bar{\mathbf{3}}$; quarks 3 & 4 are coupled identically to 1 & 2 and then the pairs $\{12\}$,$\{34\}$ coupled to a $\bar{\mathbf{6}}$ of flavour, spin 0 and a $\mathbf{3}$ of colour. This is the flavour-spin-colour structure of the Jaffe-Wilczek correlation. The antiquark couples trivially to give an overall colour singlet.

As stated this wavefunction (flavour-spin-colour-spatial) is only antisymmetric under exchange of labels $1 \leftrightarrow 2$ or $3 \leftrightarrow 4$, but not for example $1 \leftrightarrow 3$. This would seem to violate the generalized Pauli principle. In fact we need not go to the trouble of antisymmetrising two particles if their wavepackets have very limited overlap - we don't have to worry about electrons on the moon when we solve the Schrödinger equation for a hydrogen atom on Earth nor, more pertinently, do we usually antisymmetrise the quarks in different nucleons when we study the deuteron. Here we are proposing that the dynamics in the Hamiltonian given earlier will be such that the wavefunctions of quarks in different diquarks have very limited overlap such that we don't have to antisymmetrise them. We will show that this assumption is consistent when we variationally solve the Hamiltonian with this trial wavefunction.

With the $|\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c\rangle_{\mathbf{3}_c}$ colour state the expectation values of the potentials are

$$\begin{aligned}
\langle\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c|V(q^4)|\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c\rangle &= \frac{5}{3}a(\rho^2+\nu^2) + \frac{2}{3}a\lambda^2 \\
\langle\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c|V(\bar{q})|\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c\rangle &= \frac{1}{3}a(\rho^2+\nu^2) + \frac{1}{3}a\lambda^2 + \frac{4}{3}a\tau^2 \\
\langle\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c|V^\sigma(q^4)|\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c\rangle &= \frac{2}{3}h\big(\vec{S}_1.\vec{S}_2\delta(\vec{r}_{12}) + \vec{S}_3.\vec{S}_4\delta(\vec{r}_{34})\big) \\
&+ \frac{1}{6}h\big(\vec{S}_1.\vec{S}_3\delta(\vec{r}_{13}) + \vec{S}_1.\vec{S}_4\delta(\vec{r}_{14}) + \vec{S}_2.\vec{S}_3\delta(\vec{r}_{23}) + \vec{S}_2.\vec{S}_4\delta(\vec{r}_{24})\big) \\
\langle\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c|V^\sigma(\bar{q})|\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_c\rangle &= \frac{1}{3}h\,\vec{S}(q^4).\vec{S}(\bar{q})\delta(\vec{r}_{1\bar{q}}).
\end{aligned} \tag{7}$$

[2] μ is the $q^4, \bar{q}$ reduced mass, $\frac{m_{\bar{q}}4m}{4m+m_{\bar{q}}}$.

With the spatial wavefunction (6) we have

$$\begin{aligned}
\langle\delta(\vec{r}_{12})\rangle = \langle\delta(\vec{r}_{34})\rangle &= \left(\frac{\gamma}{\sqrt{2\pi}}\right)^3 \\
\langle\delta(\vec{r}_{13})\rangle = \langle\delta(\vec{r}_{14})\rangle = \langle\delta(\vec{r}_{23})\rangle = \langle\delta(\vec{r}_{24})\rangle &= \frac{1}{2}\left(\frac{\gamma}{\sqrt{2\pi}}\right)^3 \frac{\beta^5}{((\gamma^2+\beta^2)/2)^{5/2}} \\
\langle\rho^2\rangle = \langle\nu^2\rangle &= \frac{3}{2\gamma^2};\ \ \langle\lambda^2\rangle = \frac{5}{2\beta^2} \\
\langle\vec{\nabla}_\rho^2\rangle = \langle\vec{\nabla}_\nu^2\rangle &= \frac{3\gamma^2}{2};\ \ \langle\vec{\nabla}_\lambda^2\rangle = \frac{5\beta^2}{2}.
\end{aligned} \tag{8}$$

The spin structure of $\langle\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_c|V^\sigma(q^4)|\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_c\rangle$ guarantees an attractive potential only for the spin-0, spin-0 correlation,

$$\langle\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_c|V^\sigma(q^4)|\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_c\rangle = \frac{h}{6}\left(\frac{\gamma}{\sqrt{2\pi}}\right)^3 \begin{pmatrix} -6 \\ 2-\frac{\beta^5}{((\beta^2+\gamma^2)/2)^{5/2}} \\ 2+\frac{1}{2}\frac{\beta^5}{((\beta^2+\gamma^2)/2)^{5/2}} \end{pmatrix} \left\{ \begin{array}{l} 0\otimes 0 \to 0 \\ 1\otimes 1 \to 0 \\ 1\otimes 1\to 2 \end{array}\right. \tag{9}$$

Compare this with the result of the schematic approximation, eqn(1). We see that by suitable choice of C, the schematic approximation can duplicate the $0\otimes 0$ result, but that unless there is accidental cancellation it would be unable to describe the $1\otimes 1\to 0$ result. The origin of the non-zero value for $1\otimes 1 \to 0$ is the different expectation values of the delta function with a P-wave and without (see equation(8)). We propose that this is a general problem with the schematic approximation when applied to states with non-trivial spatial dependence and that by its use one can miss significant physics.

Returning to our study of the quark potential model we see that $\langle\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_c|V^\sigma(\bar{q})|\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_c\rangle$ is particularly simple. The spatial and colour dependence is identical for all quarks and factors out leaving a sum of the quark spins, which in the $0\otimes 0\to 0$ channel is zero, hence the anti-quark does not change the hyperfine energy of the system in the Jaffe-Wilczek correlation.

The expectation value of the Hamiltonian is thus,

$$\langle JW|H|JW\rangle = \left[\frac{3\alpha^2}{4\mu}+\frac{2a}{\alpha^2}\right] + \left[\frac{5\beta^2}{4m}+\frac{5a}{2\beta^2}\right] + 2\left[\frac{3\gamma^2}{4m}+\frac{3a}{\gamma^2}\right] - h\left(\frac{\gamma}{\sqrt{2\pi}}\right)^3. \tag{10}$$

The minimum is found to be at $\alpha = (8\mu a/3)^{1/4}$, $\beta = (2ma)^{1/4}$ and γ satisfying $\frac{3h}{(2\pi)^{3/2}}\gamma^5 - \frac{3}{m}\gamma^4 + 12a = 0$. We can set the parameters m, a, h using conventional baryon spectroscopy and along the way demonstrate that this model can perfectly well describe the $\Delta - N$ splitting.

a can be set approximately using the S-wave P-wave splitting (ω_P) of roughly 500MeV for the non-strange baryons, since in the harmonic oscillator $\omega_P = \sqrt{4a/m}$. This gives $a \sim \frac{(405\text{MeV})^4}{4\times 330\text{MeV}}$, where the reason for this unusual presentation will become clear later. If we consider one-gluon-exchange to be the origin of the contact term [8, 10], then $h = \frac{8\pi}{3}\frac{\alpha_S}{m^2}$, where $\alpha_S \sim 0.75$ is usual. The light quark mass takes its conventional value $m = 330$MeV.

The expectation value of the Hamiltonian for the nucleon between S-wave Gaussian trial wavefunctions (with oscillator parameter α_ρ) is then

$$2\left[\frac{3\alpha_\rho^2}{4m}+\frac{3a}{\alpha_\rho^2}\right] - \frac{1}{3}\sqrt{\frac{2}{\pi}}\frac{\alpha_S}{m^2}\alpha_\rho^3, \tag{11}$$

which is minimised by $\alpha_\rho = 440$MeV, with colour-spin hyperfine energy -150MeV. This corresponds to a $\Delta - N$ splitting of 300MeV, in good agreement with data [11] [3]. My approach differs from that usually taken which considers the hyperfine term only in first order of perturbation theory. By using a variational ansatz we are allowing the hyperfine term to modify both the energy and the wavefunction (α_ρ would have been 405MeV had the hyperfine term been neglected - hence the rather unusual form of a presented earlier which makes this obvious).

Returning to the Jaffe-Wilczek state, using these parameter values we find $\alpha = 370$MeV, $\beta = 340$MeV and $\gamma = 520$MeV. Had we neglected the hyperfine term, γ would have been 405MeV. The hyperfine term is reducing the mean distance between quarks 1 & 2 and quarks 3 & 4 relative to the $\{12\}\{34\}$ distance, which is exactly what one demands of a spatial diquark-diquark state and what we need for the unsymmetrised ansatz to be justified. Specifically with these numbers the diquarks have mean radius $\sim 1/3$ fm and are separated by an average distance ~ 1 fm.

With $\gamma = 520$MeV, the hyperfine energy is -510MeV, much larger than the cost of exciting the P-wave ($\frac{\beta^2}{m} \sim 350$MeV), so that the Jaffe-Wilczek state is considerably lighter than one would have naively expected. Simply adding together quark masses and including a P-wave energy of ~ 350 MeV would suggest a mass over 2 GeV. Consider the difference $m(\Theta) - m(N)$ which in this model (where the Hamiltonian does not include the quark rest masses) will be

$$\langle JW|H|JW\rangle + m + m_s - \langle N|H|N\rangle.$$

This is found to be (with $m_s = 450$MeV), ~ 590MeV and hence $m(\Theta) \sim 1530$MeV. This agreement is fortuitous - we have, for example, neglected tensor interactions which are non-zero due to the P-wave character and which will change this prediction.

We have observed something rather interesting - A Jaffe-Wilczek-like Θ state of low mass with compact diquarks has emerged from a model which also gives an excellent description of the $\Delta - N$ splitting. From conventional quark model analysis one would not have expected this - the usual colour-spin arguments as applied to the nucleon suggest a spin-zero "diquark" of mass ~ 600 MeV which if used naively in the Jaffe-Wilczek scheme would hugely overpredict the Θ mass. What we have demonstrated here is that the diquarks in the Θ are not like the "diquark" in the nucleon. The quarks in the nucleon all overlap spatially and correct antisymmetrisation must be carried out. This does not allow for the kind of spatially distinct diquark found for the Θ, instead the correlation can only be in spin and flavour.

3 Flavour-Spin 210 "ground state"

The Jaffe-Wilczek correlation would not be our first guess for the ground state of the $q^4\bar{q}$ system. Without colour-spin interactions a state with S-waves between all quarks would be expected to have lower energy. Jaffe & Wilczek suggest that the colour-spin interactions would force such a state to a higher energy than their correlation and it is this to which we now turn in this simple model.

The totally antisymmetric q^4 state with symmetric spatial part is in a **210** of flavour-spin and

[3] of course, the parameter α_S has been chosen to give this good agreement

is coupled to total spin $S(q^4) = 1$ [12]. It has explicit form

$$\frac{1}{\sqrt{3}}\left\{|\mathbf{6}_1,\mathbf{6}_1\rangle\otimes|\bar{\mathbf{3}}_c,\bar{\mathbf{3}}_c\rangle+\left(\frac{\sqrt{3}}{2}|\bar{\mathbf{3}}_1,\bar{\mathbf{3}}_0\rangle-\frac{1}{2}|\mathbf{6}_0,\mathbf{6}_1\rangle\right)\otimes|\mathbf{6}_c,\bar{\mathbf{3}}_c\rangle+\left(\frac{\sqrt{3}}{2}|\bar{\mathbf{3}}_0,\bar{\mathbf{3}}_1\rangle-\frac{1}{2}|\mathbf{6}_1,\mathbf{6}_0\rangle\right)\otimes|\bar{\mathbf{3}}_c,\mathbf{6}_c\rangle\right\}_{\bar{\mathbf{6}},1\otimes\mathbf{3}_c}, \tag{12}$$

where we use the "diquark" notation without implying that diquarks are dynamically generated. The spatial form (for q^4) is

$$\psi = N[Y_{00}(\hat{\lambda})e^{-\bar{\beta}^2\lambda^2/2}][Y_{00}(\hat{\rho})e^{-\bar{\beta}^2\rho^2/2}][Y_{00}(\hat{\nu})e^{-\bar{\beta}^2\nu^2/2}], \tag{13}$$

whose overall symmetry is exposed by expressing the exponent in terms of the $\vec{r}_i$ as $-2\bar{\beta}^2\sum_{i>j}(\vec{r}_i-\vec{r}_j)^2$.

In this flavour-spin-colour state the potential $V(q^4)$ is $\frac{4a}{3}(\lambda^2+\rho^2+\nu^2)$. A straightforward but somewhat lengthy computation yields the expectation of $V^\sigma(q^4)$ in this state. This is simplified slightly by the expectation of the delta function, $\langle\delta(\vec{r}_{ij})\rangle = \left(\frac{\bar{\beta}}{\sqrt{2\pi}}\right)^3$, being i,j independent. We find

$$\langle V^\sigma(q^4)\rangle = \frac{8\pi}{9}\frac{\alpha_S}{m^2}\left(\frac{\bar{\beta}}{\sqrt{2\pi}}\right)^3, \tag{14}$$

which is repulsive, as anticipated. Comparing with the q^4 part of the Hamiltonian in the Jaffe-Wilczek case we find that the $\mathbf{210}_{FS}$ is around 60MeV heavier. We have not considered the colour-spin interaction between q^4 and the anti-quark which may be non-zero due to the $S(q^4) = 1$ character of the $\mathbf{210}_{FS}$ state. This could raise the mass of the $\mathbf{210}_{FS}$ even further.

Thus in this simple model with colour-spin contact interactions a Jaffe-Wilczek-like state can be consistently defined which is lighter than the naive ground state. The diquark size is small on the scale of their separation, which was assumed in the spin-orbit analysis of [13].

4 Smearing the delta function

One should worry about the validity of using a delta function interaction in a Hamiltonian. Such an object is too singular at the origin to use the normal boundary condition $u(0) = 0$ to quantise the energy levels; we have ignored this problem by using a variational ansatz which satisfies the usual boundary condition. We still have a problem if we consider the origin of this term - it came at order $(v/c)^2$ in a non-relativistic reduction of the one-gluon-exchange process, hence in using this term we are implicitly assuming that the momenta of the quarks are much lower than their mass. At very small interquark separations the corresponding momentum scale is rather large and the non-relativistic approximation breaks down. As such we should "smear out" the delta function on the scale of the quark mass. A suitable modification is

$$\delta(\vec{r}) \to \frac{1}{(r_0\sqrt{\pi})^3}\exp[-r^2/r_0^2] \tag{15}$$

with $r_0 \sim m_q^{-1}$. This type of smearing has been used previously in the literature when modeling conventional hadron spectra [9, 14]. The following smearing scales were found to be phenomenologically satisfactory:

[9] $r_0 \sim 1/(1280\text{MeV}) \sim 1/(4m_u)$;
[14] $r_0 \sim 1/(1870\text{MeV}) \sim 1/(6m_u)$.

We will consider here the effect of smearing on the results we reported in the previous section. The change is $\langle\beta|\delta(\vec{r})|\beta\rangle \equiv (\beta/\sqrt{\pi})^3 \to (\beta/\sqrt{\pi})^3(r_0^2\beta^2+1)^{-3/2}$. With $r_0 = 1/(nm_u)$, the following is obtained for the nucleon, Jaffe-Wilczek state and the $\mathbf{210}_{FS}$

	nucleon		JW		$\mathbf{210}_{FS}$	
n	α_ρ	E_{hyp}	γ	E_{hyp}	$\bar{\beta}$	E_{hyp}
1	408	-31	410	-62	365	19
4	428	-124	464	-307	358	54
6	433	-139	483	-379	358	57
10	436	-147	501	-444	358	59

The delta-function is recovered in the limit $n \to \infty$. So while the nucleon and $\mathbf{210}_{FS}$ state are not strongly affected by the smearing for the phenomenological values $n \sim 4 \to 6$, the large effects felt by the Jaffe-Wilczek state are diluted considerably. Since the P-wave excitation energy is independent of n, the dilution is such that the Jaffe-Wilczek state is heavier than the $\mathbf{210}_{FS}$.

That the conclusions arrived at in the previous sections are cut-off dependent is disappointing, but at least the trends remain. The Jaffe-Wilczek-like state still undergoes a considerable downward shift and spatially localised diquarks still seem to be formed.

5 Extensions

As mentioned in the introduction, an attractive feature of diquark correlations is their supposed ability to describe the light scalar mesons. A simple extension to this work would consider $qq\bar{q}\bar{q}$ states; however, without a P-wave to separate the diquark from the anti-diquark it seems unlikely that spatially distinct diquarks of the type found earlier will emerge. Of course such a study would share many similarities with the classic analysis (in the bag model) of Jaffe [5].

Another interesting state to consider is the *ududud* state with two P-waves. This has the same quantum numbers as the deuteron. If three diquarks are formed with P-waves between them we would hope that it has mass greater than the measured mass of the deuteron which we know to be well described as a bound-state of a proton and a neutron with small overlap. Consideration of such a state has been advocated in [13] as a test on any model proposed to give a light Θ.

This model, extended to allow unequal quark masses, could predict the masses of the other members of the pentaquark $\overline{\mathbf{10}} \oplus \mathbf{8}$, being an explicit realisation of $SU(3)_F$ breaking. In particular this would test the phenomenological Jaffe-Wilczek Hamiltonian $H_s = M_0 + (n_s + n_{\bar{s}})m_s + n_s\alpha$ and its predictions for the other pentaquark states.

If confirmed, the anti-charmed pentaquark observed by H1 [15] with a mass of 3.1 GeV would have its mass significantly underpredicted by this model, as it is in the original Jaffe-Wilczek paper [3]. Setting the charm quark mass using the experimental $\Lambda_c - \Lambda$ mass difference and computing the new Hamiltonian expectation, we find a mass of around 2.8 GeV for the equivalent anti-charmed Jaffe-Wilczek-like state, which is within 100 MeV of the Jaffe-Wilczek prediction and very close to the DN threshold. If the magnitude of spin-orbit splitting calculated in [13] is correct, the possibility that the observed state is the $3/2^+$ state is unlikely. One possibility would be that the lightest anti-charmed pentaquark pair $(1/2^+, 3/2^+)$ is still to be found and that the H1 observation is an excited state, such as the vector diquark excitation. The state $1 \otimes 1 \to 0$ in the model presented in this paper is about 400MeV heavier than the Jaffe-Wilczek-like state.

This would be a little heavy for the H1 candidate but probably within model errors; alternatively the state with one vector diquark and one scalar diquark will be somewhat lighter and might be a possibility, but only if the H1 state is the $I_z = 0$ part of an isovector.

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